Reply to "Comment on 'Thermodynamic and transport properties of dense hydrogen plasmas' "

Ronald Redmer
Fachbereich Physik, Universität Rostock, Universitätsplatz 3, D-18051 Rostock, Germany
(Received 7 May 1997; revised manuscript received 15 August 1997)

The use of the Debye potential for the dense plasma region was criticized in a comment on a paper by Reinholz *et al.* [Phys. Rev. E **52**, 5368 (1995)]. The Debye screening model is of limited validity but other many-particle effects such as partial ionization, structure factor, local-field corrections, and degeneracy have to be considered for high densities. The electrical conductivity calculated within the present model is in reasonable agreement with available experimental data. [S1063-651X(98)16002-6]

PACS number(s): 52.25.Kn, 52.25.Fi

We agree completely with the comment of Tkachenko that the Debye model for screening in a charged particle system is of limited validity. The classical criterion that the number of particles in the so-called Debye sphere has to be greater than one is not fulfilled in the high-density domain considered in Ref. [1]. On the other hand, screening effects are not strictly limited to the Debye sphere. The potential strength of the interaction between two particles is only reduced by a factor of $1/e$ at the Debye length compared with the bare Coulomb potential due to screening. Furthermore, the treatment of the dynamics of the screening process is necessary instead of using the static Debye potential (see Refs. [2,3]). Another possibility to go beyond the simple Debye model is to solve the nonlinear Poisson equation [4] instead of the linearized version. Such a more detailed treatment of screening was not intended in Ref. [1]. A self-consistent treatment of screening and correlation effects in a fully ionized plasma was given, e.g., in Ref. [5].

In dense, low-temperature plasmas, the treatment of many-particle effects other than screening is of central importance. Therefore, in Ref. [1] we focused on the influence of partial ionization, T matrix cross sections, and the ion-ion structure factor on the thermodynamic and transport properties of dense hydrogen plasmas within a quantum statistical approach valid for arbitrary degeneracy. Only two quantities, the two-particle partition function [Eq. (9)] and the electron transport cross sections [Eq. (40)], have been evaluated using the Debye potential for the calculation of scattering phase shifts. Although important for the numerical results, this is not the keystone of our paper.

The equation of state was evaluated by taking into account exchange and correlation contributions between charged particles, the interactions with neutrals (polarization) and between neutrals (van der Waals attraction and hard-core repulsion). However, we have not interpreted the exchange and correlation contributions in terms of the dielectric function which corresponds to the self-consistent treatment of screening in Refs. [6] and [7].

On the other hand, one should note that we have taken into account bound and scattering state contributions to the two-particle partition function. The corresponding total correlated two-particle density [Eq. (9)] leads to a lower bound for the conductivity when interpreted in terms of an ionization degree [model (c) of Ref. [1]]. Considering only bound state contributions [Planck-Larkin partition function, models (a) and (b) in Ref. [1]], a higher ionization degree and, thus, a higher conductivity follows.

The conductivities calculated within the method outlined in Ref. [1] agree with available experimental data. This has been shown in detail for the Coulombic part (fully ionized plasma) and partially ionized alkali plasmas; see Refs. [8] and [9]. In addition, using an improved equation of state for dense, neutral hydrogen fluid without ionization [10], we have performed calculations for the electrical conductivity within the partially ionized plasma (PIP) model of Ref. [1] for conditions reached in recent shock-wave experiments

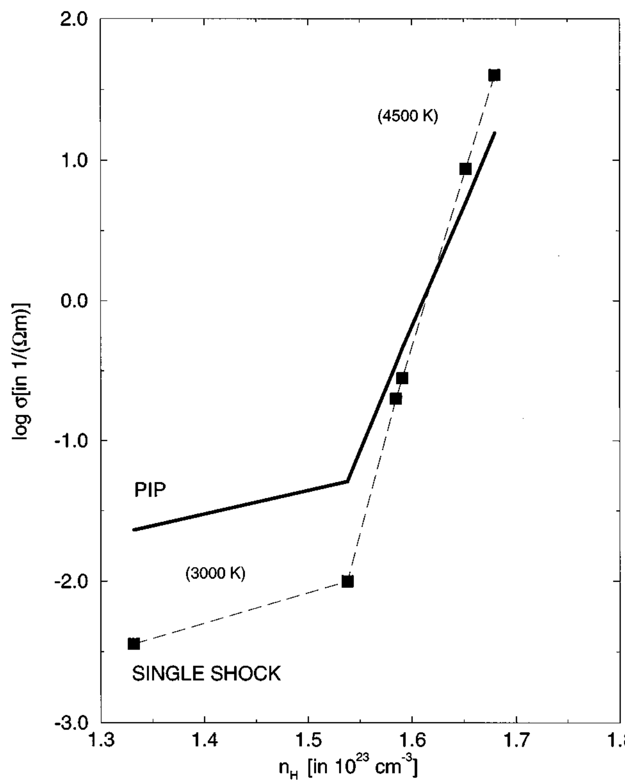

FIG. 1. Conductivity in dense fluid hydrogen measured in single-shock experiments (dashed line, Ref. [11]) compared with the partially ionized plasma (PIP) model of Ref. [1] (solid line).

[11,12]. For example, in Fig. 1 we compare our results with the single-shock data [11] between $(1-2)\times10^{23}$ cm^{-3} and $3000-4500$ K. For those conditions, the ionization degree is very low (between 3×10^{-10} and 4×10^{-6}) and the corresponding conductivities are in the range of the experimental data. In particluar, the strong increase of the conductivity with the density is reproduced. Therefore, I cannot agree with the statement that the conductivities reported in Ref. [1] are doubtful in the high-density region.

A few remarks to the comment of Tkachenko have to be added. Equation (12) in Ref. [1] gives the screening length based on the linearized Poisson equation using the Fermi distribution function for the electron density at arbitrary degeneracy of the plasma. This formula is not constructed; it yields the correct limiting cases, the Debye length for nondegenerate plasmas and the Thomas-Fermi length for degenerate plasmas. This equation has to be multiplied by 4π on the right-hand side.

Another misprint occurred in Eq. (20) of Ref. [1] defining the partition functions of rotational and vibrational states. The correct equation is

$$\sigma_{H_N}^{\rm rot}=\frac{k_BT}{hcB_{H_N}}, \tag{1}$$

$$\sigma_{H_N}^{\rm vib}=\left[1-\exp\left(-\frac{hc\omega_{H_N}}{k_BT}\right)\right]^{-1}.$$

The numerical results are not affected in either cases.

The parametrization for the local-field corrections $G(k)$ of Ichimaru and Utsumi [13] is strictly valid for $T=0$ and $r_S\leqslant15$. The numerical results have shown that the ion-ion structure factor $S_{ii}(k)$ is only important in the high-density, degenerate domain where temperature effects are negligible so that the parametrization of Ref. [13] can be applied.

Unfortunately, we missed the subsequent papers of Adamyan *et al.* [7], which give new results for the conductivity of fully ionized plasmas within a generalized Lorentz approximation [6]. Comparing the results of Ref. [1] for the fully ionized reference plasma (Fig. 7) with their results (Fig. 1 of Ref. [6]), rather good agreement can be stated. The results of Ref. [6] are obtained within a self-consistent model for the polarization function. The authors have given a low-density expansion of the Spitzer formula [Eq. (7)], which has misled us in the characterization of their numerical results.

I conclude that the conductivities calculated within the partially ionized plasma model of Ref. [1] show the right low- and high-density asymptotes for fully ionized plasmas, reproduce available experimental results for partially ionized plasmas, and yield qualitative agreement with recent shock-wave experiments in dense fluid hydrogen.

[1] H. Reinholz, R. Redmer, and S. Nagel, Phys. Rev. E **52**, 5368 (1995).

[2] H. Gould and H. E. DeWitt, Phys. Rev. **155**, 68 (1967); see also R. H. Williams and H. E. DeWitt, Phys. Fluids **12**, 2326 (1969).

[3] R. Redmer, G. Röpke, F. Morales, and K. Kilimann, Phys. Fluids B **2**, 390 (1990).

[4] J. R. Meyer, Phys. Rev. B **20**, 1762 (1979).

[5] S. Ichimaru, S. Mitake, S. Tanaka, and X.-Z. Yan, Phys. Rev. A **32**, 1768 (1985); S. Mitake, S. Tanaka, X.-Z. Yan, and S. Ichimaru, *ibid.* **32**, 1775 (1985).

[6] Z. Djurić, A. A. Mihajlov, V. A. Nastasyuk, M. M. Popović, and I. M. Tkachenko, Phys. Lett. A **155**, 415 (1991).

[7] V. M. Adamyan, Z. Djurić, A. M. Ermolaev, A. A. Mihajlov, and I. M. Tkachenko, J. Phys. D **27**, 111 (1994); **27**, 927 (1994).

[8] G. Röpke and R. Redmer, Phys. Rev. A **39**, 907 (1989).

[9] R. Redmer, Phys. Rep. **282**, 35 (1997).

[10] A. Bunker, S. Nagel, R. Redmer, and G. Röpke, Phys. Rev. B **56**, 3094 (1997).

[11] W. J. Nellis, A. C. Mitchell, P. C. McCandless, D. J. Erskine, and S. T. Weir, Phys. Rev. Lett. **68**, 2937 (1992).

[12] S. T. Weir, A. C. Mitchell, and W. J. Nellis, Phys. Rev. Lett. **76**, 1860 (1996).

[13] S. Ichimaru and K. Utsumi, Phys. Rev. B **24**, 7385 (1981).